



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 39

American Options – 2

J P Singh

Department of Management Studies



AMERICAN OPTIONS: PUT CALL PARITY

	$t=0$	$t=\tau$	$t=T$	
PORTFOLIO A		$S_\tau < K$	$S_T < K$	$S_T > K$
BUY EUROPEAN CALL	$-c$	≥ 0	0	$S_T - K$
INVEST	$-K$	$Ke^{(r\tau)}$	$Ke^{(rT)}$	$Ke^{(rT)}$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$

	$t=0$	$t=\tau$	$t=T$	
PORTFOLIO A		$S_\tau < K$	$S_T < K$	$S_T > K$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$
PORTFOLIO B				
BUY STOCK	$-S_0$	S_τ	S_T	S_T
BUY AMERICAN PUT	$-P$	$K - S_\tau$	$K - S_T$	0
TOTAL	$-(S_0 + P)$	K	K	S_T
$S_0 + P \leq c + K$ but $c = C$, so $S_0 - K \leq C - P$				

From previous slide: $S_0 - K < C - P$

$$c + Ke^{(-rT)} = p + S_0$$

$$c - p = S_0 - Ke^{(-rT)}$$

$$c = C; p \leq P \text{ and } c, p > 0$$

$$C - P \leq c - p$$

$$C - P \leq S_0 - Ke^{(-rT)}$$

$$S_0 - K \leq C - P \leq S_0 - Ke^{(-rT)}$$

WHAT ABOUT $S_t > K$??



PUT CALL PARITY WITH DIVIDENDS

- As shown in the case without dividends, when there are no dividends $C - P \leq S_0 - Ke^{-rT}$
- Dividends reduce C and increase P. **Why??**
- Hence relationship must also be true when there are dividends.



PUT CALL PARITY WITH DIVIDENDS

- As shown in the case without dividends, when there are no dividends $C - P \leq S_0 - K_e^{-rT}$
- Dividends reduce C and increase P. **Why??**
- Hence relationship must also be true when there are dividends.



AMERICAN OPTIONS: PUT CALL WITH DIVIDENDS

	$t=0$	$t=\tau$	$t=T$	
PORTFOLIO A		$S_\tau < K$	$S_T < K$	$S_T > K$
BUY EUROPEAN CALL	$-c$	≥ 0	0	$S_T - K$
INVEST	$-(K + D_0)$	$Ke^{(r\tau)} + D_\tau$	$Ke^{(rT)} + D_T$	$Ke^{(rT)} + D_T$
TOTAL	$-(c + K + D_0)$	$\geq (Ke^{(r\tau)} + D_\tau)$	$Ke^{(rT)} + D_T$	$S_T + Ke^{(rT)} + D_T - K$

	$t=0$	$t=\tau$	$t=T$	
PORTFOLIO A		$S_\tau < K$	$S_T < K$	$S_T > K$
TOTAL	$-(c+K+D_0)$	$\geq (Ke^{(r\tau)} + D_\tau)$	$Ke^{(rT)} + D_T$	$S_T + Ke^{(rT)} + D_T - K$
PORTFOLIO B				
BUY STOCK	$-S_0$	$S_\tau + D_\tau$	$S_T + D_T$	$S_T + D_T$
BUY AMERICAN PUT	$-P$	$K - S_\tau$	$K - S_T$	0
TOTAL	$-S_0 - P$	$K + D_\tau$	$K + D_T$	$S_T + D_T$

Since $c=C$, we have $S_0 + P \leq C + K + D_0$ so $S_0 - D_0 - K \leq C - P \leq S_0 - Ke^{-rT}$

EARLY EXERCISE OF AMERICAN CALL ON DIVIDEND PAYING STOCK

- We have already shown that in the absence of dividends American options should never be exercised early.
- It can be similarly argued that, when there are dividends, it can only be optimal to exercise at a time immediately before the stock goes ex-dividend.



- Hence, we shall consider the possibility of early exercise only on the dividend dates i.e. when the stock goes ex-dividend.
- We consider the case when only one dividend, D , is expected in $(0,T)$, say at $t=T_d$ at which point the stock will go ex-dividend. Hence, we have to examine the possibility of early exercise at T_d .



- If the option is exercised at $t=T_d$ the payoff received by the option holder is $S(T_d)-K$.
- If the option is not exercised, the stock price will drop down to $S(T_d) - D$.
- With this stock price, the lower bound on the option price will be $[S(T_d) - D] - K \exp[-r(T-T_d)]$.
- Thus, if $[S(T_d) - D] - K \exp[-r(T-T_d)] > S(t_d) - K$, it is surely not optimal to exercise the option i.e. if $D < K\{1 - \exp[-r(T-T_d)]\}$ it cannot be optimal to exercise the call.

- On the other hand, if $D > K\{1 - \exp[-r(T - T_d)]\}$ it may or may not be optimal to exercise the call.
- However, for any reasonable assumption about the stochastic process followed by the stock price, it can be shown that it is always optimal to exercise at time T_d for a sufficiently high value of $S(T_d)$.
- The $>$ inequality will tend to be satisfied when the ex-dividend date is fairly close to the maturity of the option (i.e., $T - T_d$ is small) and the dividend D is large.



EXAMPLE

- An Indian trader constituted a portfolio today ($t=0$) comprising of
 - (i) a long 180-day European call covering 100 USD with a strike of $\text{INR } 72 = \text{USD } 1$ and
 - (ii) a short position in a 180-day forward contract for 100 USD.
- The premium on the call was $\text{INR } 20.00$.

- The spot and 180-day forward INR/USD exchange rates were respectively INR 70=USD 1 & INR 75=USD 1.
- The trader expects the exchange rate on maturity to take either of only two values viz. INR 71 = USD 1 and INR 74 = USD 1 with probabilities of 0.75 & 0.25 respectively.
- Ignoring time value of money, calculate the expected profit from this portfolio (in INR) at maturity.

	SCENARIO A	SCENARIO B
SPOT PRICE AT MATURITY (INR/USD)	71	74
STRIKE PRICE	72	72
CALL PAYOFF	0	200
CALL PRICE	20	20
NET PAYOFF ON CALL	-20	180
SPOT COST OF 100 USD	7100	7400
NET COST OF 100 USD	7120	7220
SELLING PRICE PER USD	75	75
SALES PROCEEDS OF 100 GBP	7500	7500
NET PROFIT	380	280
PROBABILITY	0.75	0.25
EXPECTED PROFIT		355